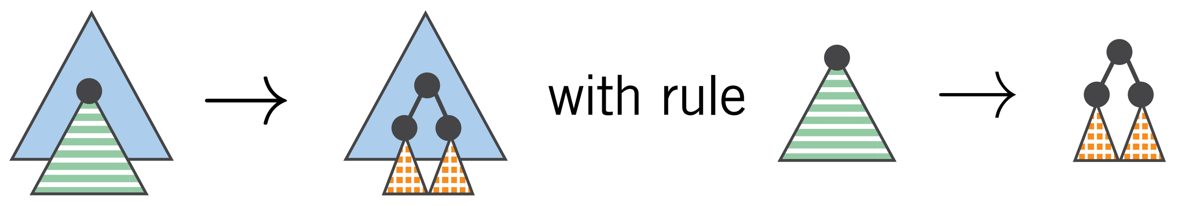
From Trees to Graphs: Understanding the Implications of Sharing for Rewriting

Motivation

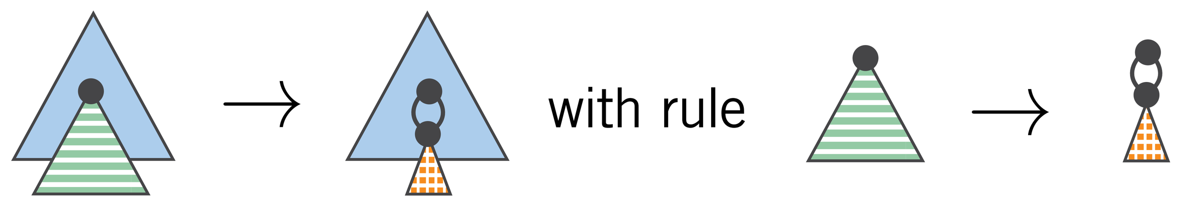
- rewriting = transformation of objects based on rules
- term rewriting [1,2] objects are terms = trees

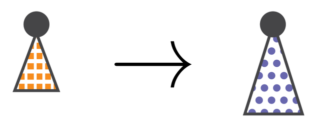


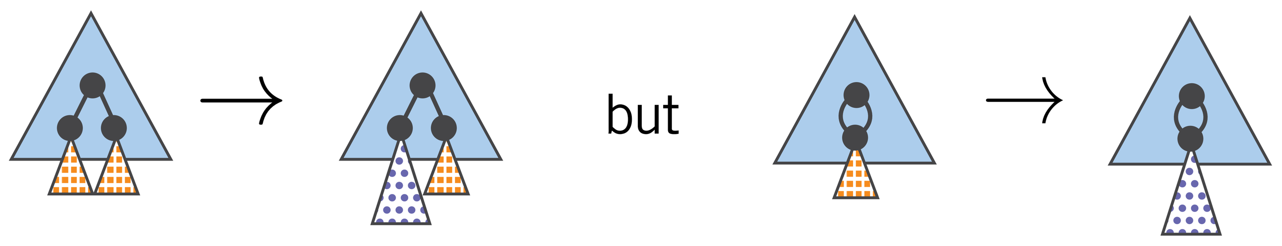
term
f(x,x)
=
f
x x

term graph [3]
f:①
x:②

- from trees to graphs ~ term graph rewriting
  sharing of equal subterms, avoids blow-up in size

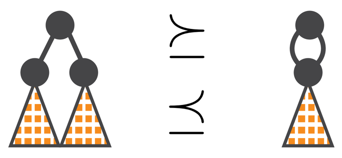


- influences the potential rewrite steps with rule: 

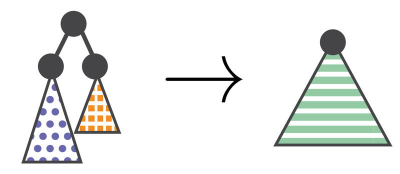


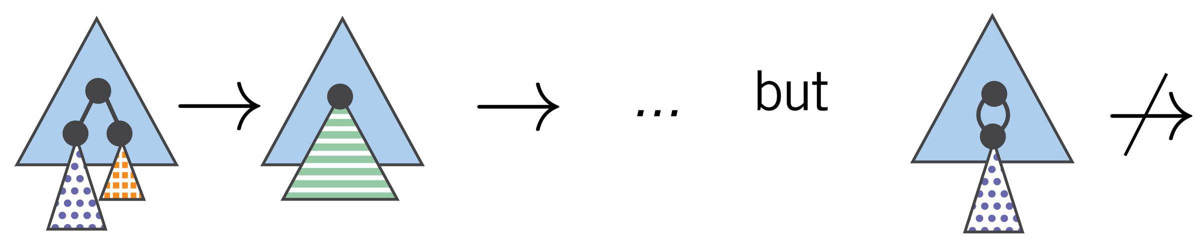
Aim: understand the implication of sharing on rewriting & on termination, i.e., the absence of infinite rewrite sequences

Background & Related Work

- term graph rewriting with explicit sharing **and** unsharing 
  simulates term rewriting with linear size growth and polynomial overhead [3,4]
- but $\preceq$ counter-intuitive, hence investigate with only $\succeq$

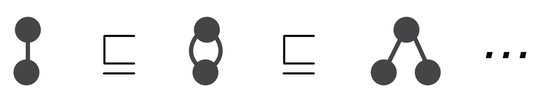
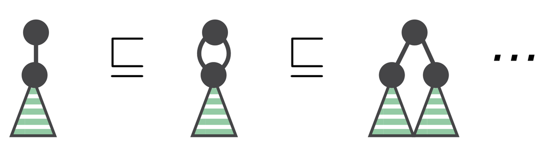
Every term graph rewrite step can be simulated by n term rewrite steps.

Termination of term rewriting
$\Rightarrow$ terminination of term graph rewriting [5]
$\not\Leftarrow$ see infinite rewrite sequence with rule: 

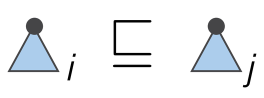
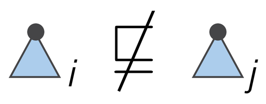


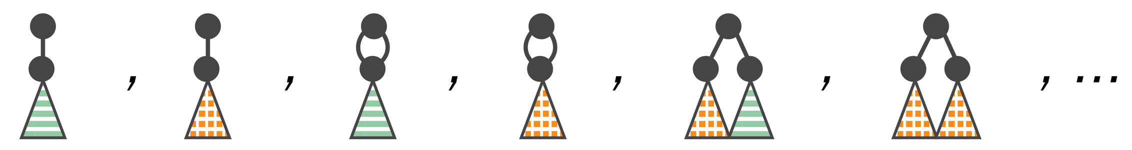
- show termination of term graph rewriting after [6]
  re-prove result & transfer to formalism in [3]

Contribution

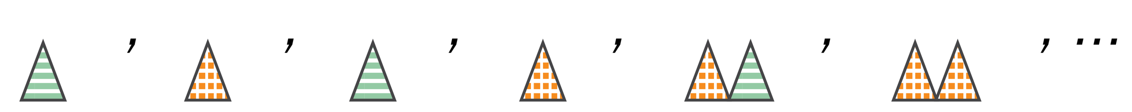
Theorem: A well-quasi order $\sqsubseteq$ on **Top** 
can be extended to
a well-quasi order $\sqsubseteq$ on term graphs 

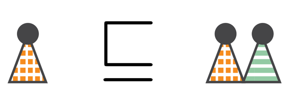
Proof Sketch: (Kruskal's tree theorem [7], minimal bad sequences [8])

- $\sqsubseteq$ is a well-quasi order if all infinite sequences are "good"
  "good" means for some i < j: 
- construct minimal "bad" sequence **T**
  "bad" means for all i < j : 

T: 

- take arguments of **T**

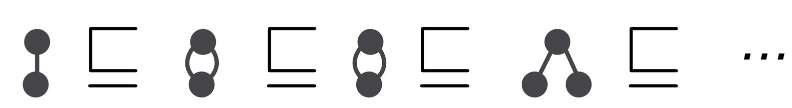
G: 

by minimality of **T** and transitivity of $\sqsubseteq$, **G** is "good",
i.e., there is a 

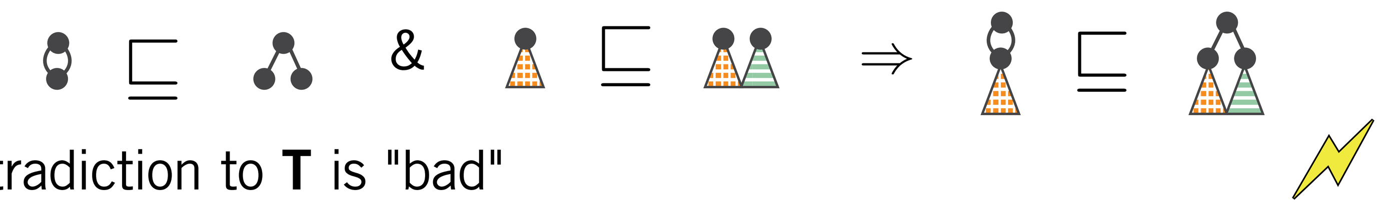
- take Tops of **T**

f: 

there exists an infinite subsequence such that

fφ: 

- construct from Top and argument graph



contradiction to **T** is "bad"

News:

- definition of $\sqsubseteq$ for term graph flavour [3]
- re-prove directly with Kruskal's tree theorem
- insight: view arguments as *one* argument graph

...and so what?

Vision

- basis for a termination order on term graphs [6]
- basis for an automated termination analysis for term graph rewriting
- applications for term graph rewriting

______________________?

Summary: By moving from a tree to a graph representation, the termination behavior of rewriting changes. I re-proved that an order on the top of term graphs can be extended to an order on term graphs. This is the basis for constructing a termination order and enabling automated termination analysis.

References:
[1] Baader, F, Nipkow, T: Term Rewriting and All That (1998)
[2] TeReSe (2003)
[3] Avanzini, M: Verifying Polytime Computability Automatically (2013)
[4] Kennaway, JR, et al: On The Adequacy Of Graph Rewriting For Simulating Term Rewriting (1994)
[5] Plump, D: Term Graph Rewriting (1999)
[6] Plump, D: Simplification Orders for Term Graph Rewriting (1997)
[7] Kruskal, JB: Well-Quasi-Ordering, The Tree Theorem, and Vazsonyi's Conjecture (1960)
[8] Nash-Williams, CSJA: On Well-Quasi-Ordering Finite Trees (1963)

Maria A Schett is currently a master student of Computer Science at the University of Innsbruck, Austria. So far, her research interests are (i) automatically analysing termination and complexity of graph rewriting, and (ii) constructing compilers with higher-order rewriting.

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